

Boolean algebra, as developed in 1854 by George Boole in his book *An Investigation of the Laws of Thought*. Boolean algebra is the algebra of truth values 0 and 1. *The operations are usually taken to be conjunction, disjunction, and negation, with constants 0 and 1. Applications include mathematical logic, digital logic, computer programming, set theory, and statistics.*

*Boolean algebra is used to design digital circuits. Boolean A circuit is designed by connecting AND, OR, NOT gates together. Gates can be composed into complex circuits of electronic devices.*

<p><i>“I will take an umbrella with me if it is raining or the weather forecast is bad AND not take the car”</i></p>	
<p><i>(Take Umbrella) = ( NOT (Take Car ) ) AND ( (Bad Forecast ) OR (Raining ) )</i></p>	

**Basic Gates used in Circuit Design**

<p><b>AND</b></p> <p>AND ·</p> <table border="1" style="font-size: small;"> <thead> <tr><th>A</th><th>B</th><th>R</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	A	B	R	0	0	0	0	1	0	1	0	0	1	1	1	<p><b>OR</b></p> <p>OR +</p> <table border="1" style="font-size: small;"> <thead> <tr><th>A</th><th>B</th><th>R</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	A	B	R	0	0	0	0	1	1	1	0	1	1	1	1	<p><b>NOT</b></p> <p>NOT'</p> <table border="1" style="font-size: small;"> <thead> <tr><th>A</th><th>R</th></tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td></tr> </tbody> </table>	A	R	0	1	1	0	<p><b>NAND</b></p> <p><math>(A \cdot B)'</math></p> <p><b>NOR</b></p> <p><math>(A + B)'</math></p>
A	B	R																																					
0	0	0																																					
0	1	0																																					
1	0	0																																					
1	1	1																																					
A	B	R																																					
0	0	0																																					
0	1	1																																					
1	0	1																																					
1	1	1																																					
A	R																																						
0	1																																						
1	0																																						
<p><b>XOR</b></p> <p><math>Q = A \oplus B</math></p> <table border="1" style="font-size: small; margin: 10px;"> <thead> <tr><th>A</th><th>B</th><th>XOR</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>				A	B	XOR	0	0	0	0	1	1	1	0	1	1	1	0																					
A	B	XOR																																					
0	0	0																																					
0	1	1																																					
1	0	1																																					
1	1	0																																					

### Boolean Rules to know ...

#### *Basic Boolean algebraic identities*

Additive	Multiplicative
$A + 0 = A$	$0A = 0$
$A + 1 = 1$	$1A = A$
$A + A = A$	$AA = A$
$A + \bar{A} = 1$	$A\bar{A} = 0$

#### *Basic Boolean algebraic properties*

Additive	Multiplicative
$A + B = B + A$	$AB = BA$
$A + (B + C) = (A + B) + C$	$A(BC) = (AB)C$
$A(B + C) = AB + AC$	

#### *Useful Boolean rules for simplification*

$$A + AB = A$$

$$A + \bar{A}B = A + B$$

$$(A + B)(A + C) = A + BC$$

### De Morgan's Law



$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

[http://en.wikipedia.org/wiki/Augustus\\_De\\_Morgan](http://en.wikipedia.org/wiki/Augustus_De_Morgan)

## Boolean Arithmetic

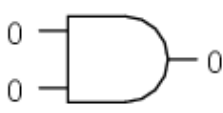
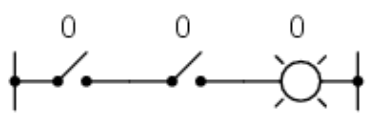
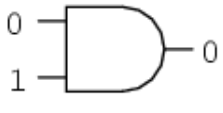
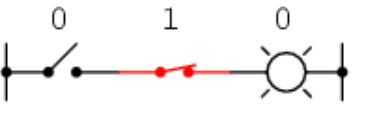
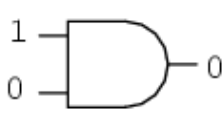
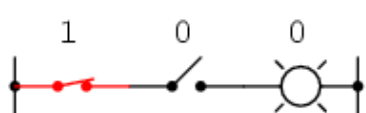
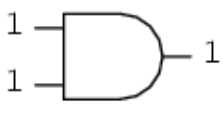
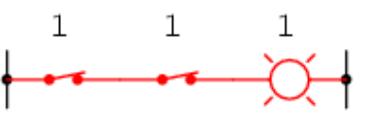
### Fundamentals of addition:

$0 + 0 = 0$	$0 + 1 + 1 = 1$	<i>No matter how much we add 1s the output is 1</i>
$0 + 1 = 1$	$1 + 1 + 1 = 1$	
$1 + 0 = 1$	$0 + 1 + 1 + 1 = 1$	
$1 + 1 = 1$	$1 + 0 + 1 + 1 + 1 = 1$	


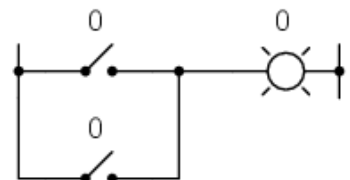

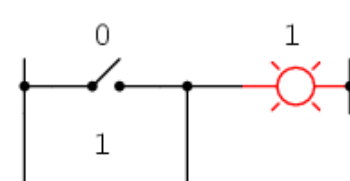

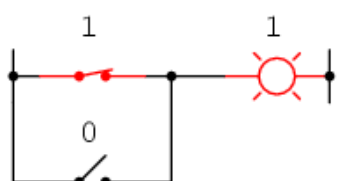

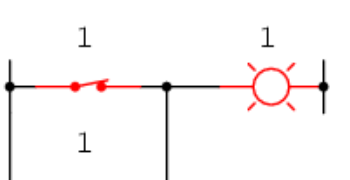
### Multiplications work like this

$0 \times 0 = 0$	<i>There is no subtraction or division in Boolean algebra.</i>
$0 \times 1 = 0$	
$1 \times 0 = 0$	
$1 \times 1 = 1$	

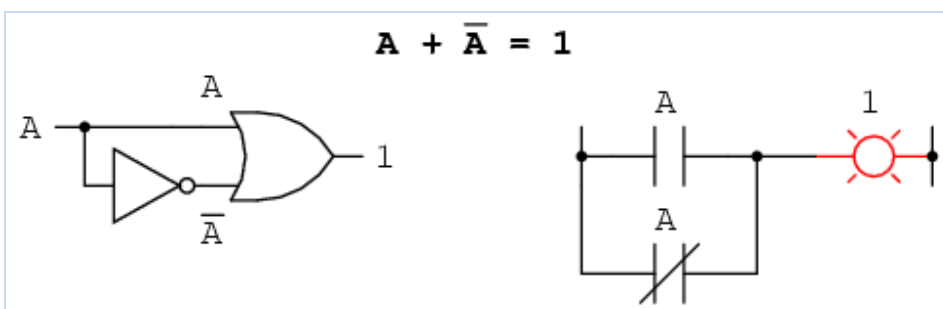
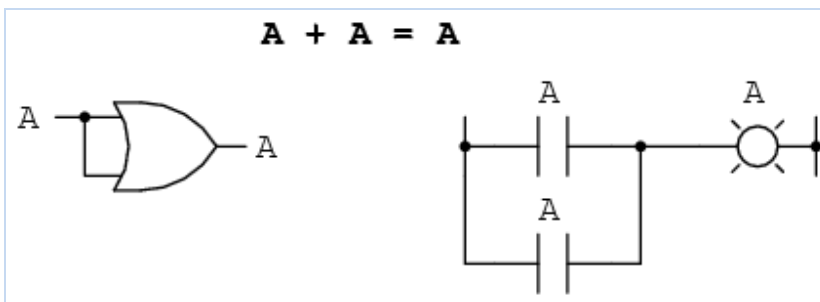
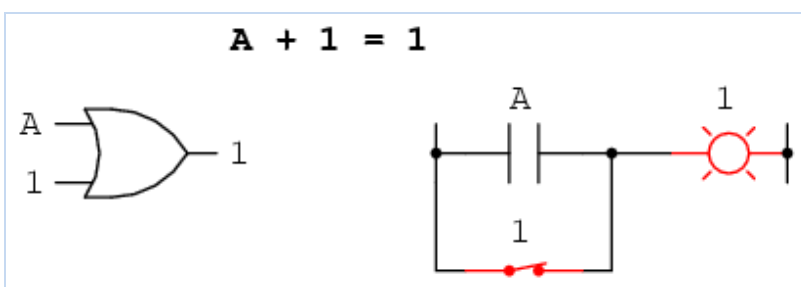
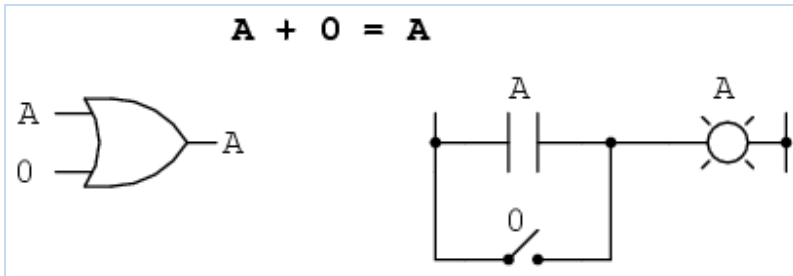
**AND gate: (Serial Switches)**

<p style="text-align: center;"><math>0 \times 0 = 0</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>	<p><b>AND .</b></p> <table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">A</th> <th style="padding: 5px;">B</th> <th style="padding: 5px;">R</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> </tr> </tbody> </table>	A	B	R	0	0	0	0	1	0	1	0	0	1	1	1
A		B	R													
0		0	0													
0		1	0													
1	0	0														
1	1	1														
<p style="text-align: center;"><math>0 \times 1 = 0</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>																
<p style="text-align: center;"><math>1 \times 0 = 0</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>																
<p style="text-align: center;"><math>1 \times 1 = 1</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>																

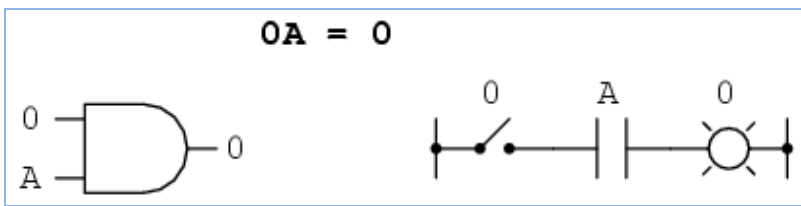
**OR gate: (parallel switches)**

<p style="text-align: center;"><math>0 + 0 = 0</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>	<p style="text-align: center;"><b>OR +</b></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">A</th> <th style="padding: 5px;">B</th> <th style="padding: 5px;">R</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> </tr> </tbody> </table>	A	B	R	0	0	0	0	1	1	1	0	1	1	1	1
A		B	R													
0		0	0													
0		1	1													
1	0	1														
1	1	1														
<p style="text-align: center;"><math>0 + 1 = 1</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>																
<p style="text-align: center;"><math>1 + 0 = 1</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>																
<p style="text-align: center;"><math>1 + 1 = 1</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>																

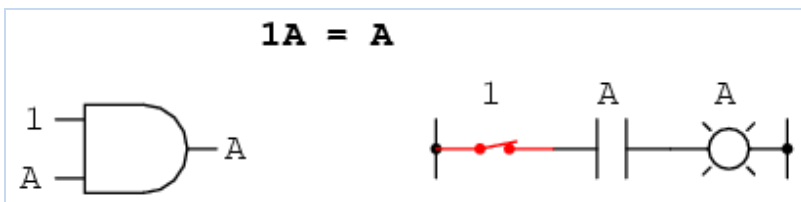
**OR gates ...**



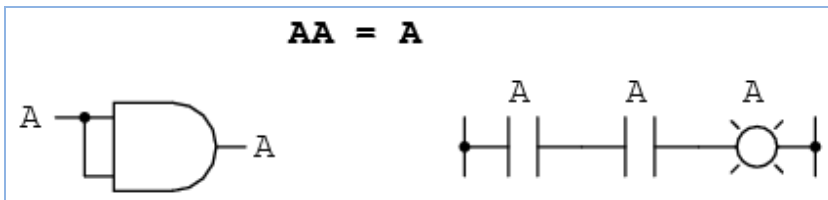
**$0A = 0$**



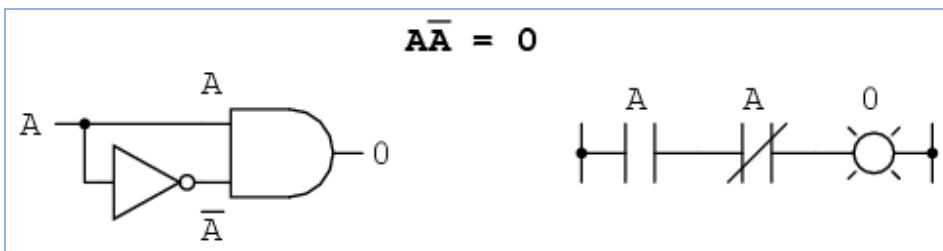
**$1A = A$**



**$AA = A$**



**$A\bar{A} = 0$**



*Basic Boolean algebraic identities*

**Additive**

**Multiplicative**

$A + 0 = A$

$0A = 0$

$A + 1 = 1$

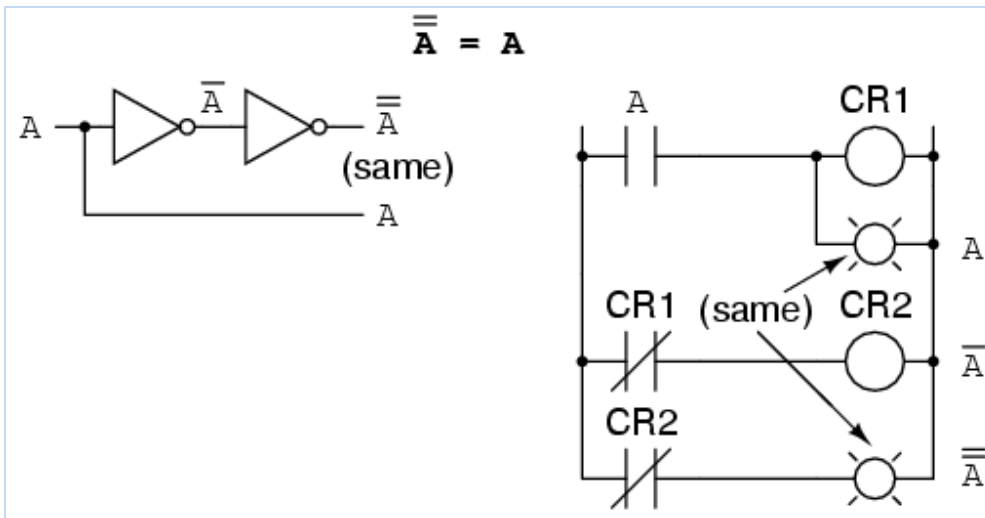
$1A = A$

$A + A = A$

$AA = A$

$A + \bar{A} = 1$

$A\bar{A} = 0$



*Niranjana Meegamma (2011)*

<http://advanced-level-ict.blogspot.com>

[www.shilpasayura.org](http://www.shilpasayura.org)